A
Correlation Between Expansion and Elastic Modulus

This appendix includes MPR Calculation 0326-0062-CLC-03, *Correlation Between Through-Thickness Expansion and Elastic Modulus in Concrete Test Specimens Affected by Alkali-Silica Reaction (ASR)*, Revision 2.

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

CALCULATION TITLE PAGE

Client: NextEra Energy Seabrook	Page 1 of 12 + Appendix A and B (29 pages total)
Project: Approach for Estimating Through-Wall Expansion from Alkali-Silica Reaction at Seabrook Station	Task No. 0326-1405-0074
Title: Correlation Between Through-Thickness Expansion and Elastic Modulus in Concrete Test Specimens Affected by Alkali-Silica Reaction (ASR)	Calculation No. 0326-0062-CLC-03

Preparer / Date	Checker / Date	Reviewer & Approver / Date	Rev. No.
Michael Saitta February 2, 2015	Vaibhav Bhide February 2, 2015	John W. Simons February 2, 2015	0
Michael Saitta June 23, 2015	Kathleen Mulvaney June 23, 2015	John W. Simons June 23, 2015	1
Amanda E. Card July 19, 2016 (Page 1 to 12+Appendix A)	Keith Means July 19, 2016 (Page 1 to 12 +Appendix A)	John W. Simons July 19, 2016	2
Keith Means July 19, 2016 (Appendix B)	Amanda Card July 19, 2016 (Appendix B)		2

QUALITY ASSURANCE DOCUMENT

This document has been prepared, checked, and reviewed/approved in accordance with the QA requirements of 10CFR50 Appendix B and/or ASME NQA-1, as specified in the MPR Nuclear Quality Assurance Program.

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

RECORD OF REVISIONS

Calculation No.	Prepared By	Checked By	Page: 2
0326-0062-CLC-03	Amanda Card	Keith Means	

Revision	Affected Pages	Description
0	All	Initial Issue
1	All	Added correction factor for through-thickness expansion values to account for influence of mid-plane cracks on the expansion measured using embedded rods.
2	All	Added final test results, updated figures, and revised correlation equation.

Note: *The revision number found on each individual page of the calculation carries the revision level of the calculation in effect at the time that page was last revised.*

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: 3
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

Table of Contents

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: 4
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

1.0 PURPOSE

This calculation determines a correlation between through-thickness expansion and normalized elastic modulus of concrete test specimens affected by Alkali-Silica Reaction (ASR). The correlation is based on data from test programs that MPR sponsored at Ferguson Structural Engineering Laboratory (FSEL). The correlation is compared to published data.

2.0 SUMMARY OF RESULTS

There is a strong correlation between elastic modulus and through-thickness expansion of concrete specimens that are affected by ASR. The data were fit with a least squares regression using a [redacted] form. Figure 2-1 below shows the FSEL test data and the least squares fit. The least squares fit compares favorably with the trend observed in the data. The R^2 value of the correlation is [redacted]. Figure 2-1 also shows data found in the literature for free expansion of ASR-affected concrete specimens. These data are consistent with the FSEL data.

[redacted]

Figure 2-1. Strong Correlation between Elastic Modulus and Through-Thickness Expansion

3.0 BACKGROUND

Published data show that the material properties of ASR-affected concrete change with increasing levels of ASR-related expansion. MPR will use the relationship between material properties and ASR-related expansion to determine the through-thickness expansion of concrete structures at Seabrook Station.

This relationship is defined using data from test programs that MPR sponsored at FSEL to investigate ASR in reinforced concrete elements. The test specimens were consistent with structures at Seabrook Station in terms of reinforcement details, depth of cover, and overall depth. In addition, the concrete used in the test specimens was representative of the concrete used at Seabrook Station, with some deviations to produce significant ASR-related expansion in a short timeframe.

4.0 ASSUMPTIONS

4.1 Assumptions with a Basis

There are no assumptions with a basis.

4.2 Unverified Assumptions

There are no unverified assumptions.

5.0 DISCUSSION

5.1 Test Data

The test data used herein are for test specimens from the Shear Test Program and the Reinforcement Anchorage Test Program, as well as the Instrumentation Test Program. Combining data from these three programs is appropriate as the same concrete mix was used in all test specimens. In addition, the test specimen configurations and reinforcement details were similar (Reference 6).

Data from all ASR-affected test specimens are used in this calculation. This includes data from [redacted] test specimens: [redacted] reinforcement anchorage specimens, [redacted] shear specimens, and the instrumentation beam.

The baseline material properties are the 28-day tests performed on cylinders molded at the time of concrete placement. The material properties at various levels of ASR-related expansion are based on tests of cores removed from the test specimen prior to structural testing. The data include the following:

- 28 days after concrete placement (before ASR-related expansion occurred)
  - Three compressive strength values,
  - Three elastic modulus values, and
  - Three splitting tensile strength values[1].
- Prior to structural testing (after ASR-related expansion occurred)
  - Three compressive strength values,
  - Three elastic modulus values,
  - Three splitting tensile strength values[2], and
  - Through-thickness expansion values.

All values are taken from MPR-4262 (Reference 6) or Reference 8 and are summarized in Appendix A.

5.2 Selection of Elastic Modulus as the Property for the Correlation

To facilitate comparisons, the material properties of each test specimen from the post-ASR cores were normalized against its average value from the 28-day cylinders. Therefore, a sample that had seen very little change in a material property would have a normalized value of approximately 1, whereas one that had experienced a 25% reduction in a material property would have a normalized value of 0.75.

Figure 5-1 plots the normalized compressive strength and the normalized elastic modulus versus through-thickness expansion. From the plot, it appears that there is a strong correlation between modulus and through-thickness expansion. There also appears to be a weak correlation between compressive strength and through-thickness expansion.

There were insufficient data to normalize the splitting tensile strength. Therefore, the splitting tensile strength was plotted against through-thickness expansion in Figure 5-2. There does not

[1] Note that 28-day results for splitting tensile strength are not available for specimens that were cast before May 2014 ([redacted]) (Reference 6).

[2] Note that the test programs did not start performing splitting tensile testing until the end of May 2014. Therefore, test-day splitting tensile strength test results are not available for [redacted] Procedure 5-6 allows omission of splitting tensile tests on cores due to the difficulty in extracting testable cores from members with significant cracking due to ASR. Using this provision, splitting tensile strength testing was not performed on cores from [redacted] Similarly, only two cores from [redacted] and one core from [redacted] were tested (Reference 6).

appear to be a correlation between splitting tensile strength and expansion. Therefore, it is determined that elastic modulus is the best choice to correlate against expansion.

[redacted]

Figure 5-1. Normalized Compressive Strength and Modulus vs. Through-Thickness Expansion

[redacted]

Figure 5-2. Splitting Tensile Strength vs. Through-Thickness Expansion

5.3 Elastic Modulus Correlation

Non-linear least squares regression was used to fit a curve for the correlation between normalized modulus and expansion. Based on scoping analysis of several types of equations (e.g. natural log, exponential, power, etc.), it was determined that the best-fit curve would take the form of:

[redacted]

Least squares fitting was used to determine the constants A and B. The process of least squares is described in detail in Appendix B. This resulted in a final correlation of:

[redacted]

Where:

expansion is the relative through-thickness expansion of the concrete specimen (0.02 implies a 2% expansion) and
modulus is the normalized modulus of the test specimen after ASR.

This correlation is shown in below in Figure 5-3. The least squares fit compares favorably with the observed data. The R^2 value for the correlation is [redacted].

[redacted]

Figure 5-3. Normalized Modulus vs. Through-Thickness Expansion: Test Data

5.4 Comparison to Published Values

Data on the elastic modulus as a function of ASR-related expansion are available in the literature. These data are for free expansion of small concrete specimens. Table 5-1 lists data from the sources considered in Reference 2.

Table 5-1. Existing Data Showing Expansion and Corresponding Elastic Modulus

Expansion (%)	Normalized Elastic Modulus (%)	Reference
0.05	100	3, Table 2.1
0.10	70	3, Table 2.1
0.25	50	3, Table 2.1
0.50	35	3, Table 2.1
1.00	30	3, Table 2.1
1.50	20	3, Table 2.1
0.002	100	4
0.039	66.0	4
0.114	65.2	4
0.210	54.7	4
0.328	50.2	4
0.392	46.7	4
0.007	100	4
0.020	97.7	4
0.038	91.2	4
0.095	78.3	4
0.128	75.8	4
0.29[1]	86.5[2]	5
1.253[1]	13.9[2]	5
0.43[1]	70.2[2]	5
1.573[1]	13.7[2]	5
0.43[1]	39.7[2]	5
1.656[1]	10.3[2]	5
0.43[1]	32.8[2]	5
1.686[1]	8.1[2]	5

Note 1: Longitudinal prism expansion was selected as the most representative.

Note 2: Taken as elastic modulus at testing divided by elastic modulus at 28 days.

Figure 5-4 plots these data and compares them to the FSEL data and to the correlation based on the FSEL data.

[redacted]

Figure 5-4. Normalized Modulus vs. Through-Thickness Expansion: Published Literature

As shown in Figure 5-4, the data from published literature follow a trend that is consistent with the FSEL test data and the correlation determined using these data.

6.0 References

1. Not Used.

2. Bayrak, Oguzhan, *Structural Implications of ASR: State of the Art*, July 28, 2014, transmitted to Seabrook Station in MPR Letter 0326-0058-200, dated July 29, 2014.

3. Clark, L.A., *Critical Review of the Structural Implications of the Alkali Silica Reaction in Concrete*, Transport and Road Research Laboratory Contractor Report 169, July 1989.

4. Smaoui, N. et al., *Mechanical Properties of ASR-Affected Concrete Containing Fine or Coarse Reactive Aggregates*, Journal of ASTM International, Vol. 3, No. 3, March 2006.

5. Ahmed, T. et al., *The effect of Alkali Reactivity on the Mechanical Properties of Concrete*, Construction and Building Materials, 17 (2003) 123-144, January 9, 2002.

6. MPR-4262, "Shear and Reinforcement Anchorage Testing of Concrete Affected by Alkali-Silica Reaction," Volume I, Revision 1 & Volume II, Revision 0. (Seabrook FP#100994)

7. MPR-4259, "Commercial Grade Dedication Report for Seabrook ASR Shear, Reinforcement Anchorage and Instrumentation Testing," Revision 0. (Seabrook FP # 100995)

8. Special Test and Inspection Reports (STIRs) as accepted by CGAR-0326-0062-43-2 Revision 0, CGAR-0326-0062-43-5 Revision 1, and CGAR-0326-0062-43-7 Revision 0.

   a) STIR-0326-24-103

   b) STIR-0326-24-104

   c) STIR-0326-24-105

   d) STIR-0326-24-147

   e) STIR-0326-24-204

   f) STIR-0326-24-228

9. MPR-4286, "Supplemental Commercial Grade Dedication Report for Seabrook Test Programs," Revision 0. (Seabrook FP# 101003)

A
Test Data

This Appendix includes tables of summarized test data originally from FSEL. Table A-1 contains data from tests conducted 28 days after casting. The data are used to normalize the post-ASR data. Table A-2 contains data from tests that were conducted after ASR had occurred (i.e., post-ASR data). Table A-3 contains the through-thickness expansion values. Test data are taken from Reference 6 or the main body of this calculation unless otherwise noted. Applicable Special Test Inspection Records (STIRs) are listed for reference.

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: A-2
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

Table A-1. FSEL 28-Day Compressive Strength, Elastic Modulus, and Splitting Tensile Strength Test Data

[redacted]	[redacted]			[redacted]			[redacted]		
	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

Table A-1. FSEL 28-Day Compressive Strength, Elastic Modulus, and Splitting Tensile Strength Test Data

[redacted]	[redacted]			[redacted]			[redacted]		
	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

MPR			MPR Associates, Inc. 320 King Street Alexandria, VA 22314
Calculation No. 0326-0062-CLC-03	Prepared By Amanda Card	Checked By Keith Means	Page: A-4 Revision: 2

Table A-2. FSEL Average Expansion, Compressive Strength, and Elastic Modulus: Test Data After ASR

[redacted]	[redacted]			[redacted]			[redacted]		
	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: A-5
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

Table A-2. FSEL Average Expansion, Compressive Strength, and Elastic Modulus: Test Data After ASR

[redacted]	[redacted]			[redacted]			[redacted]		
	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: A-6
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

Table A-2. FSEL Average Expansion, Compressive Strength, and Elastic Modulus: Test Data After ASR

[redacted]	[redacted]			[redacted]			[redacted]		
	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

MPR

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: A-7
0326-0062-CLC-03	Amanda Card	Keith Means	Revision: 2

Table A-3. FSEL Expansion Test Data With Correction Factor

[redacted]	[redacted]	
	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]

[redacted]

B

Least Squares Regression

Purpose

This appendix explains the methodology used to perform the Least Squares Regression Analysis. A brief description of the fit statistic R^2 is also given. After the method of Least Squares is explained, the method is applied to the correlation between the FSEL test data for normalized elastic modulus and corrected through thickness expansion.

Discussion

Least Squares Regression is a commonly accepted method of fitting a curve to a set of scattered data. This is done by minimizing the sum of squares error term. This is a common statistical method that is documented in textbooks such as "Applied Data Analysis and Modeling for Energy Engineers and Scientists" by T.A. Reddy. The sum of squares is given by:

$$S = \sum_{i=1}^{m} r_i^2$$

Where:

S is the error term,
m is the number of known values, and
r_i is the residual of the ith value, as given by:

$$r_i = y_i - f(x_i, C)$$

Where:

y_i and x_i are a known value pair,
f is the regressed or fit function, and
C is the set of constants used to fit the model.

By combining the above equations with a known set of values, S is minimized by varying C. In some cases, this can be accomplished analytically, but is often accomplished numerically. The values of C that minimize S are said to be the fitting parameters, and the function $f(x_i, C)$ is the curve of best fit in the least squares sense.

It is often desirable to determine how well a given curve fits a set of data. A commonly used statistic to determine this is the coefficient of determination, R^2. R^2 is defined as:

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}$$

$$SS_{res} = \sum_{i=1}^{m} (y_i - f(x_i, C))^2 = S$$

$$SS_{tot} = \sum_{i=1}^{m} (y_i - \bar{y})^2$$

Calculation

The least squares regression performed in the main body of this calculation is described in detail below. The set of points is listed in Table B-1 and plotted in Figure B-1.

Table B-1. Known Values

[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]

Table B-1. Known Values

[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]

Table B-1. Known Values

[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]
[redacted]	[redacted]

[redacted]

Figure B-1. Plot of Known Values

It appears that a natural log fit is reasonable. Therefore, it can be fit to an equation of form:

[redacted]

Where:

x is the set of values of X as shown in Table B-1.
A and B are a set of constants (C) used to fit the model.

To begin, we will guess at the values of A and B. In this example, our first guess will be that $A = -0.1$ and $B = -0.5$. Using the model given above, we compute a value for y at each given x. For each computed value, the residual is also computed. These values are shown in Table B-2.

Table B-2. Example Values With Computed Residuals

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

Table B-2. Example Values With Computed Residuals

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

Table B-2. Example Values With Computed Residuals

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

Taking the sum of squares of the residuals, we find a value of approximately [redacted] However, this can be improved on. To do so, we iteratively adjust the values A and B to minimize S.

[redacted] result in S being minimal and provide a good estimate of the solution. The fitted curve is plotted against the data in Figure B-2. The newly computed values are shown in Table B-3. The regressed equation is:

[redacted]

Table B-3. Example Values With Computed Residuals - Updated

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

MPR Associates, Inc.
320 King Street
Alexandria, VA 22314

Calculation No.	Prepared By	Checked By	Page: B-8
0326-0062-CLC-03	Keith Mears	Amanda Card	Revision: 2

Table B-3. Example Values With Computed Residuals - Updated

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

Table B-3. Example Values With Computed Residuals - Updated

[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]
[redacted]	[redacted]	[redacted]	[redacted]

[redacted]

Figure B-2. Regressed Curve

R^2 can now be computed using the regressed curve. The sum of squared residuals is [redacted] (SS_{res}). The mean of y is [redacted]. Therefore, the sum of squared totals is [redacted] (SS_{tot}). R^2 can now be computed.

$$R^2 = 1 - SS_{res}/SS_{tot}$$

[redacted]